RESEARCH AND PRACTICAL APPLICATIONS

From cubits to qubits: metrology and sensing in the ancient, modern, and quantum eras

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Quantum cables and quantum minicomputer prototype. Conceptualization: Luisa Quiroga

Quantum sensing and metrology is one of the four canonical pillars of quantum technology, alongside quantum computation, quantum simulation, and quantum communication. Sensing is the measurement of the fleeting phenomena that surround us, the extension of our natural senses by technological means. Metrology is usually defined as “the science of measurement”, a name that conceals more than it reveals. As we shall see, metrology is both an ancient practice implicated in the rise and fall of civilizations, and a technology at the cutting edge of current research. In recent decades, sensing and metrology have made increasing use of quantum physics, giving rise to quantum sensing and quantum metrology. In this essay I will sketch for a non-specialist audience the nature of these topics, and the unique role played by quantum physics in sensing and metrology.

Embodiment of length in ancient Egypt: a royal cubit standard from the tomb of Maya, the treasurer of Tutankhamen. Copyright Alain Guilleux – Fotos Egypte (photosegypte.com)

Measurement in and of ancient civilization: the cubit

A stele fragment known as the Palermo stone shows that a unit of length, the cubit, was in use in ancient Egypt, at least since the 27th century BCE, to measure the flood level of the Nile. This date is truly ancient, not long after the invention of written language. The name cubit we get from Latin, but this same measure, under different names, was used in Mesopotamia, the Levant, Egypt, and ancient Rome. In the Islamic world, the cubit was used well into the medieval period, more than three thousand years after its first recorded use in Egypt. It outlasted the Egyptian civilization, its cities, its gods, its laws and its language.

A cubit is roughly the distance from the elbow to the tip of the middle finger. It is a convenient measure, one that can readily be used for simple commerce: if you ask a merchant for two cubits of cloth, you can easily check if this is in fact what you are given. It is easy to imagine a pre-technological society getting by with informal measures like these. But the civilizations of Mesopotamia, Egypt and the Levant needed much better measurements. They built monumental structures, traded goods with distant cities, and governed large territories. To do this, they required precise measurements, and they needed them to be uniformly applied by large numbers of people.

Klenovsky et al. [1] provide a vivid and succinct description of how this may have worked:

> About 3,000 years BCE, the Egyptian unit of length, the royal cubit, was decreed to be equal to the length of the forearm – measured from the bent elbow to the tip of the extended middle finger – plus the width of the palm of the hand of the pharaoh. A master was carved from a block of black granite to endure for all time. To disseminate this unit of length, the workers building tombs, temples and pyramids were supplied with cubit sticks made of wood or granite. The royal architect or foreman of each construction site was responsible for transferring and then maintaining the unit of length to the workers' cubit sticks, which had to be compared to the royal cubit master every full moon. Failure to do so was punished by death. [2].

This illustrates several key elements of metrology as it is currently practised:

- First, there is the unit, in this case the royal cubit. This was a distinct unit from the common cubit used in the marketplace. The royal cubit is divided into seven palms, each of which is divided into four fingers.

- Second, there is the definition of the unit: the royal cubit is the length of the forearm plus the width of the palm of the pharaoh. Once defined, one can measure something in royal cubits, using the pharaoh as a reference. This is hugely impractical, since the pharaoh will be used in this way only once.

- Third, there is the primary standard, the master black granite block, cut to match the length of the pharaoh's forearm plus palm. This manufactured object embodies the royal cubit and is made of the hardest material, to make it durable and tamper-proof. The primary standard is unique and, if it is damaged or lost, the whole system fails.

- Fourth, there are the secondary standards, which are copies of the primary standard. These are used for measurements in the field, are made of lesser materials, and may degrade. This is acceptable, because more secondary standards can be made by copying the primary standard.

- Fifth, there is a procedure for calibration of the secondary standards to the primary standard. Inaccurate ones are discarded, and new secondary standards are made as needed.

- Sixth, there are administrative procedures, with enforcement provisions for non-compliance [3].

As we can see from the above, metrology has mathematical, physical and administrative dimensions. There were also personal, political and religious dimensions. The Egyptian pharaoh was revered as a god, the primary standard used him as its physical reference, and the secondary standards were copied from the primary standard. This proximity to the divine ruler made the cubit standards venerable objects, much as the garments worn by a saint are venerable.

> > Metrology is both an ancient practice involved in the rise and fall of civilizations and a technology at the forefront of current research

The greatness of a ruler gave greater legitimacy to the standard, and a ruler's greatness can be inferred by the reach of the ruler's standardization. The Akkadian emperor Sargon the Great succeeded in standardizing the cubit across Mesopotamia. This helped to unite his empire, enrich his people through commerce, and make his empire easier to govern. Metrology and empire are allies, and often suffer the same fate: Sargon the Great's cubit standard fell into disuse after the Akkadian Empire dissolved.

Crisis and rebirth: enlightenment and revolutionary metrology

Through the millennia, empires and their metrological standards came and went, with little change from the ancient Egyptian model. In Europe, the last emperor to impose a uniform set of standards was Charlemagne, around 800 CE. After Charlemagne, imperial power waned and smaller kingdoms proliferated. Local rulers defined standards to suit their aims, often to extract more tribute from their subjects. On the eve of the French Revolution, it is estimated that about 800 different measurement units were in use in France, with a quarter of a million different local definitions [4]. Commerce in these conditions was exceedingly difficult. In effect, the political system had corrupted metrology for its own ends.

The ancient system of units, like the Old Regime political system, was seen by the French revolutionaries as illegitimate, deeply unfair to the common people, and in need of a wholescale replacement. Following the storming of the Bastille in 1789, Louis XVI convened the Estates-General to negotiate with the people's representatives. Most villages included in their list of demands the establishment of uniform measures. In 1790, the French National Assembly, the *de facto* ruling body during the French Revolution, voted to create a new measurement system, what would become the International System of Units (SI), the metric system that we use today.

The new system drew upon proposals from Enlightenment thinkers of the previous century. These aimed to be rational and universal, to reform measurement "for all people, for all time", and to place metrology on an enduring foundation, safeguarded from the vicissitudes of politics. The people would no longer suffer under rules invented by pharaohs and twisted by emperors and aristocrats. These were the key elements:

- Units were digital (related by powers of ten), and were as few as possible, with the metre being the reference for other units, e.g., a litre is defined as a cubic decimetre.
- Units were defined by universally available, natural standards. A kilogram was the weight of a litre of pure water. The metre was one ten millionth the distance from the pole to the equator, along the Paris meridian.
- Primary and secondary standards and calibration were defined.
- The governance of the metric system would be international, by treaty.

Although the metric system eliminated divine monarch-derived units like the cubit or the *pied du roi* (Charlemagne's foot) and has a dry, secular nomenclature, it nonetheless smuggles in a divine element in the unit definitions. Many of the Enlightenment scientists who designed the metric system subscribed to Deism, a rationalist theology positing a non-interventionist creator whose existence is revealed through its creation: the natural world. A metre or kilogram, referenced to the Earth or to pure water, was again a measure linked to divinity.

Materialization of length after the French Revolution: public meter pattern, Place Vendôme, Paris.

The messy reality: post-revolutionary metrology

The universalist plans of the revolutionary system proved difficult to implement. The metre was the key to the metric system and was defined in terms of the size of the Earth. Measuring the meridian was difficult: the 1792 expedition of Delambre and Méchain ended in the death of Méchain, and the production of a primary standard, the mètre des Archives, from Méchain’s faulty data. Secondary standards were produced and distributed. Some were installed in the walls of buildings, where any citizen could have access to them. The image above shows a surviving wall-mounted secondary standard in Paris.

Survey work to measure the length of the meridian continued through the 19th century, including a spectacularly daring expedition by the 20-year-old North Catalan Francesc Aragó [5]. Understanding of the shape of the Earth also advanced. These refinements led to fluctuating estimates of the length of the meridian and endless debates – the usual business of science. It became clear the Earth standard would not lead to the ideal measurement “for all people, for all time” that was envisioned in 1790. In 1867, the Earth standard was effectively abandoned when the international metre was defined to be the length of the mètre des Archives, known to inaccurately reflect the Earth’s dimension, but nonetheless a perfectly functional primary standard already in use.

The spread of the metric system was aided by the Napoleonic wars, which destabilized European politics and afforded opportunities for rational-minded reformers. Once again, metrology and empire were allies. Ironically, French civilians resisted the new system that was created for their liberation, reverting for a while to traditional measures. Eventually, though, the metric system spread to nearly every corner of the globe. The greatest promise of the revolutionaries remained unfulfilled, however. The basis of the system was now the artefacts kept in Paris, primary standards that enjoyed no universality and relied upon treaties – and hence on politics – for their legitimacy and use.

A gift from microscopic physics: quantum metrology

A fundamental problem for revolutionary metrology is the mutability of the classical world. The metric system was defined using the Earth and pure water, due to their perceived immutability. But the dimensions of the Earth are altered by tides, seismic activity, volcanism and so forth. Moreover, the rotation speed of the Earth – used to define the second – is not constant. Any metrology based on the Earth will necessarily have limited precision.

Another basic problem is accessibility. Whereas pure water can be made in any location with modest resources, measurements of the Paris meridian must be made in France and Spain, and at great cost and effort. While in retrospect the Earth may have been a poor choice of reference, in fact classical physics does not offer any good choices. The pre-1900 physics of Newton, Lagrange, Maxwell, among others, describes continuous quantities that can take on any value. This physics is silent about whether these values might change in time, or from place to place. If the density of water was in fact the same everywhere, this was a purely empirical observation; there is nothing in classical physics to say it should be so.

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The discovery since 1900 of atoms and sub-atomic particles such as electrons, protons and neutrons brought new hope for universal metrology. These microscopic objects obey quantum, not classical, physics. The microscopic world is discrete: integer numbers of atoms and particles occupy a countable number of states with specific, well-defined energies. Microscopic physics is also uniform: all atoms of a given element and isotope are identical, with unchanging properties [6]. What if, rather than using artefacts as standards, metrology used atoms? Atomic spectral lines were known already in the 19th century, and proposals for their use as standards were made as early as 1827. Twentieth-century advances in spectroscopy and the development of quantum theory put such proposals on a solid foundation.

In 1960, the SI metre was defined to be exactly 1650763.73 wavelengths of the light emitted by the isotope krypton-86 on the $2p_{10}$-$5d_5$ transition [7]. It was indeed practical to measure lengths with light from specific atomic transitions, and later with light from lasers that are tuned to be resonant with atomic transitions. Later, the second was defined to be exactly 9192631770 oscillations of the Caesium (Cs) atom on its ground-state hyperfine transition. Caesium atomic clocks – oscillators tuned to the caesium transition frequency – became the best timekeepers and were sold commercially. The promise of measurement “for all people, for all time” was coming true. Still, the kilogram stubbornly resisted this approach, although efforts were made to count the number of atoms in spheres of pure crystalline silicon, for a possible definition of the kilogram as the mass of a specific number of silicon atoms.

$$1\ \text{metre} = \frac{9\,192\,631\,770}{299\,792\,458}\ \frac{c}{\Delta\nu_{Cs}}$$

Disembodiment of length since 1983: definition of the metre in terms of the speed of light *c*, the hyperfine transition
frequency of atomic caesium $\Delta\nu_{Cs}$ and a rational fraction.

The final frontier: metrology from fundamental constants

Starting in 1887, experiments by Michelson and Morley had shown that the speed of light was a constant, even for an observer in motion. Einstein built the very successful theory of special relativity around this fact, including the famous $E = mc^2$, where c is the speed of light. Relativity and its constant c became a bedrock of physical theory, and techniques to measure c grew in sophistication and precision, aided by the invention of the laser. Metrologists saw an opportunity in this: rather than defining the metre in terms of a specific atomic wavelength, they could define the metre in terms of the speed of light and the measured second. In 1983, the metre was defined this way, as illustrated above. The speed of light is now a defined quantity: exactly 299,792,458 metres per second. All present “measurements of the speed of light” are in fact measurements of the length of the metre.

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> technology exists then.

If atomic unit definitions made metrology available “for all people” through the easy availability of identical atoms, definition in terms of fundamental constants contributes to making metrology available “for all time”. Any given atomic reference, like the ^{86}Kr wavelength, is likely to be superseded by more precise or otherwise more attractive metrological techniques. By referencing the units to the fundamental constants that appear in the description of any atomic measurement, the metric system becomes stable in the face of technological change. This is the dream of a future-proof metrology system, one that can be used, not only by existing human civilizations with current technologies, but also by any future civilizations, with whatever technology they have then. In 2018, the SI system was again revised, such that now all length, time, mass, electrical and thermal units are defined in terms of the fundamental constants c (speed of light), e (electron charge), h (quantum of action), k_B (Boltzmann constant, relating temperature to energy), and $\Delta\nu_{Cs}$, the only remaining atom-referenced quantity in the system.

Quantum sensing

In parallel to these metrological developments, and benefitting from them, myriad quantum technologies for sensing have emerged. A variant of an experiment performed by Michelson and Morley to measure the speed of light is now used to detect gravitational waves (2017 Nobel Prize in Physics). The quantum physics that allows us to understand atomic spectral lines is used to design atomic instruments for exceedingly precise magnetic measurements, with applications in medical imaging [8].

Quantum information is also used in quantum sensing. Since 1980, it has been known that quantum entanglement, an enabler of quantum computation using qubits (not cubits), also enables more sensitive measurement. Since 2010, gravitational wave detectors have improved their sensitivity using entangled states known as squeezed light, also possible with atomic sensors [9].

A sufficiently advanced metrological instrument can also become, in effect, a sensor. According to Einstein’s general theory of relativity, a clock’s rate is affected by gravity. Today’s atomic clocks are so accurate that comparing two of them can enable us to measure centimetre-scale differences of depth within the Earth’s gravitational potential, and hence minute changes in the shape of the Earth.

REFERENCES AND FOOTNOTES

1 — Klenovsky, P.; Wouters, M.; De Waal, W. (2022). “The metrology behind trade”. *Nature Physics*, no. 18, p. 842.

2 — This quote is taken from a commentary written by three metrologists (to my knowledge, none of them an Egyptologist). The commentary provides no citations, but appears to draw from an opinion piece by De Bièvre, P. (2005). “Learning lessons from Ancient Egypt”. *Accreditation and Quality Assurance*, no. 10, pp. 325-326, also a metrologist. For an archeological discussion of the same question, see Monnier, F.; Petit, J.-P.; Tardy, C. (2016). “The use of the ‘ceremonial’ cubit rod as a measuring tool. An explanation”. *Journal of Ancient Egyptian Architecture*, no. 1, p. 1-9.

3 — Punishment by death is not a common feature of modern metrology schemes, but commercial metrology, for example, the measurement of electrical power, fuel, etc., often entails legal consequences for non-compliance

4 — Palaiseau, Jean-François-Gaspard (1816). *Métrologie universelle, ancienne et moderne : ou Rapport des poids et mesures des empires, royaumes, duchés et principautés des quatre parties du mond*. Bordeaux: Lavigne jeune.

5 — Aragó’s autobiography can be found online.

6 — Although quantum physics allows for atomic properties such as the mass of the electron to change over time, this change would manifest, not just in one element, but in all of them, since a single quantum theory describes all atoms. Moreover, the effects would be observable; comparing the spectrum of a many-electron atom against the spectrum of the single-electron atom hydrogen, for example. Such comparisons have been made with great precision, and no change in atomic properties has been observed.

7 — Baird, K. M.; Howlett, L. E. (1963). “The International Length Standard”. *Applied Optics*, no. 2, p. 455-463.

8 — Boto, E. et al. (2017). “A new generation of magnetoencephalography: Room temperature measurements using optically-pumped magnetometers”. *NeuroImage*, no. 149, p. 404-414.

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